

Self-consistent fluctuating hydrodynamics simulations of thermal transport in nanoparticle suspensions

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We report on the mesoscopic simulation of heat conduction in nanoparticle suspensions (nanofluids) by using the energy-conserving dissipative particle dynamics (DPD) method. Through coarse graining, our simulations probe the thermal and momentum transport in nanofluids at a length scale much greater than that in atomistic methods. We show that our simulations model the fluctuating hydrodynamics in nanofluids in a thermodynamically self-consistent manner, which is critical for resolving the current controversies on mechanisms of heat conduction in nanofluids. Simulation results indicate that the Brownian motion of nanoparticles plays a negligible role in determining the thermal conductivity of nanofluids at least within the framework of fluctuating hydrodynamics at mesoscales. © 2008 American Institute of Physics. [DOI: [10.1063/1.2908217](https://doi.org/10.1063/1.2908217)]

INTRODUCTION

Nanofluids are a new type of heat transfer fluids consisting of nanoparticles dispersed in base fluids. Despite the considerable scattering in the data reported by different groups, many experiments demonstrated that the thermal conductivity of base fluids can be dramatically enhanced by adding a small amount of nanoparticles.^{1–4} Such dramatic enhancement is difficult to explain by using the classical effective medium theories [e.g., the Maxwell–Garnett (MG) theory⁵]. Many alternative theories attributed the anomalously high thermal conductivity of nanofluids to the clustering and/or Brownian motion of nanoparticles inside the base fluids.^{6–10} Here, we focus on the potential role of particle Brownian motion in enhancing the heat conduction in nanofluids.

While the Brownian motion of nanoparticles has been proposed to enhance the thermal transport in nanofluids via a variety of ways (see Ref. 7), it is generally thought that the Brownian motion of nanoparticles could induce a microscopic convection inside an otherwise stationary fluid and, thus, greatly enhancing the thermal transport. Such a concept has been a center of controversies in nanofluids research. While it has been rejected by scaling analysis,¹¹ sophisticated semiempirical and statistical theories have been developed based on such a concept and they appear to be supported by the favorable comparison of theoretical prediction and experimental data.^{7,12} Settling the present controversies by experimental study is difficult as key input parameters of the existing theories such as the interfacial thermal resistance between fluids and nanoparticles and the mesostructure of nanoparticles in base fluids, which are now recognized to play a critical role in thermal transport in nanofluids, are difficult to experimentally quantify.¹³

Numerical simulation can be very useful in resolving the current controversies since it eliminates the ambiguity in the

parameters of nanofluids. Most of the published molecular dynamics (MD) simulations suggest that the effects of nanoparticle Brownian motion on thermal transport is minor.^{11,14–16} A major concern of these studies, however, is that in MD simulations, the size of the nanoparticle and the system is extremely small (the nanoparticles typically have a diameter of less than 3 nm) and, quite often a significant heat flux must be imposed to compute the thermal conductivity, which causes a large density heterogeneity in the nanofluids. As such, these simulation results have met considerable reservation in the engineering community. Such reservation is also due to, at least in part, that other numerical simulations support the significance of nanoparticle Brownian motion. For example, recent direct numerical simulation of fluctuating hydrodynamics equations in nanofluids and lattice Boltzmann simulations both suggest that the Brownian motion of nanoparticles can significantly enhance the thermal transport in nanofluids.^{17–19} However, in these latter simulations, the Brownian motion of nanoparticles is modeled in a very crude fashion and the fluctuation-dissipation theorem was not rigorously satisfied. Thus, it is not clear whether the significant enhancement observed in these simulations is in fact due to the nonrigorous treatment of the particle Brownian motion.

Ideally, to resolve the above controversies by numerical simulation, the following requirements should be met: (1) The size of the nanoparticle should be comparable to that used in most experiments, i.e., on the order of 10 nm. (2) The Navier–Stokes equations for thermal and momentum transport should be satisfied. (3) The Brownian motion of nanoparticles and microconvection in base fluids need to be modeled in a thermodynamically self-consistent manner, i.e., the fluctuation-dissipation theorem must be satisfied. Meeting these requirements amounts to the rigorous modeling of fluctuating hydrodynamics in a system spanning 10–1000 nm in dimension. Performing a simulation meeting such requirements is challenging because modeling moving particles in fluids is highly nontrivial and incorporating thermal fluctuations that rigorously satisfies the fluctuation-

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dissipation theorem is even more difficult.²⁰ Indeed, this represents an important gap in computational fluid dynamics, i.e., while the computational methods at both atomistic and macroscopic scales are becoming mature, there are relatively few efficient methods at the mesoscopic scale. In fact, simulation of nanofluids meeting all the above three requirements has not been reported in literature yet.

In this research, we investigate the heat conduction in nanofluids by using a mesoscopic modeling tool, the energy-conserving dissipative particle dynamics (DPD) method. We show that our simulations satisfy the above requirements and the simulation results indicate that, within the framework of fluctuating hydrodynamics, the Brownian motion of nanoparticles does not noticeably affect the heat conduction in nanofluids. The rest of this paper is organized as follows: the simulation method is presented first, which is followed by a description of the simulation systems. Validation of the simulation method and codes is discussed next, and the simulation results and conclusions are then presented.

DPD METHOD

DPD is a particle method developed for the simulation of complex fluids at the mesoscales.^{21,22} While the original DPD method is limited to isothermal applications, the recent energy-conserving DPD can handle thermal transport.^{23,24} The DPD method described here refers to this extended construct. Unlike particles in MD simulations, particles in DPD (called “beads”) are coarse-grained entities characterized by their mass, heat capacity, position, velocity, and temperature (signifying the internal energy). DPD systems evolve according to²³

$$d\mathbf{r}_i = \mathbf{v}_i dt, \quad (1)$$

$$m d\mathbf{v}_i = \mathbf{F}_i^C dt + \mathbf{F}_i^D dt + \mathbf{F}_i^R \sqrt{dt}, \quad (2)$$

$$c_v dT_i = q_i^{\text{visc}} dt + q_i^{\text{cond}} dt + q_i^{\text{rand}} \sqrt{dt}, \quad (3)$$

where m , c_v , \mathbf{r}_i , \mathbf{v}_i , and T_i are the mass, heat capacity, position, velocity, and temperature of particle i , respectively. DPD particles interact with each other via three types of forces: the conservative force \mathbf{F}^C , the dissipative force \mathbf{F}^D , and the random force \mathbf{F}^R . DPD particles also exchange energy with each other via three types of heat fluxes: the conduction heat flux q^{cond} , the random heat flux q^{rand} , and the viscous heating q^{visc} . The forces and heat fluxes acting on bead i are given by²³

$$\mathbf{F}_i^C = \sum_{j \neq i} a_{ij} w(r_{ij}/r_c) \mathbf{e}_{ij}, \quad (4)$$

$$\mathbf{F}_i^D = \sum_{j \neq i} -\gamma_{ij} w_d^2(r_{ij}/r_c) (\mathbf{e}_{ij} \cdot \mathbf{v}_{ij}) \mathbf{e}_{ij}, \quad (5)$$

$$\mathbf{F}_i^R = \sum_{j \neq i} \sigma_{ij} w_d(r_{ij}/r_c) \theta_{ij} \mathbf{e}_{ij}, \quad (6)$$

$$q_i^{\text{cond}} = \sum_{j \neq i} \kappa_{ij} w_e^2(r_{ij}/r_c) (1/T_i - 1/T_j), \quad (7)$$

$$q_i^{\text{rand}} = \sum_{j \neq i} \alpha_{ij} w_e(r_{ij}/r_c) \theta_{ij}^{\prime}, \quad (8)$$

$$q_i^{\text{visc}} = \frac{1}{2C_v} \sum_{j \neq i} w_d^2(r_{ij}/r_c) [\gamma_{ij} (\mathbf{e}_{ij} \cdot \mathbf{v}_{ij})^2 - \sigma_{ij}^2/m] - \sigma_{ij} w_d(r_{ij}/r_c) (\mathbf{e}_{ij} \cdot \mathbf{v}_{ij}) \theta_{ij}, \quad (9)$$

where a_{ij} is the conservative force coefficient and $r_{ij} = |\mathbf{r}_{ij}| = |\mathbf{r}_i - \mathbf{r}_j|$. w , w_d , and w_e are the weighting functions with cutoff distances of r_c . $\mathbf{e}_{ij} = \mathbf{r}_{ij}/r_{ij}$, and $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$. θ_{ij} and θ_{ij}^{\prime} are symmetric and antisymmetric random variables with zero mean and unit variance, respectively. The variables γ_{ij} and σ_{ij} determine the strength of the dissipative and random forces, and the variables κ_{ij} and α_{ij} determine the strength of the conduction and random heat fluxes. The fluctuation-dissipation theorem requires that

$$\gamma_{ij} = \sigma_{ij}^2 (T_i + T_j) / 4k_B T_i T_j, \quad (10)$$

$$\alpha_{ij}^2 = 2k_B \kappa_{ij}, \quad (11)$$

where k_B is the Boltzmann constant. Typically, σ_{ij} is taken as a constant and $\kappa_{ij} = c_v^2 \kappa_{0,ij} (T_i + T_j)^2 / 4k_B$. It has been shown that, with such a choice of κ_{ij} , the conduction heat flux between them is proportional to the temperature difference between the two particles and shows the least dependence on the absolute value of the temperature. In DPD, the mass, length, energy, and time are measured by m , r_c , $k_B T$, and $\sqrt{k_B T/m}$, respectively. The physical length and time scales probed in DPD depends on the mapping between DPD and physical scales, and this will be discussed below.

The DPD method has been successfully used in studying self-assembly, polymer dynamics in shear flow, and heat conduction in heterogeneous media, to name just a few.^{25–28} These studies, along with the statistical mechanics analysis of the DPD model,^{29,30} have established that the DPD model not only produces the correct hydrodynamic and thermal transport behavior in complex geometries but also, by construction, incorporates thermal fluctuations that satisfy the fluctuation-dissipation theorem. In essence, the DPD method can be regarded as a particle-based solver that solves the fluctuating hydrodynamics equations in a thermodynamically self-consistent manner.

MODELING OF PARTICULATE SUSPENSIONS

In the DPD models for a particulate suspension, base fluids are modeled as regular DPD beads and solid particles are modeled by bonding regular DPD beads together, which is similar to that in MD simulations. The key issue in DPD simulations is the choice of the simulation parameters and the mapping of the DPD system to a physical system. The key parameters of a DPD model include the DPD bead density ρ and temperature T , a_{ij} , σ_{ij} , $\kappa_{0,ij}$, r_c , and functional forms of the weighting functions w , w_d , and w_e . ρ controls the level of coarse graining, and r_c is the unit length in the DPD model and is typically taken as 1.0. The other parameters control the thermophysical properties of the base fluids and nanoparticles. Specifically, a_{ij} controls the thermodynamic properties of the system (e.g., isothermal compress-

ibility and self-assembly behavior), σ_{ij} controls the hydrodynamic properties (i.e., fluid viscosity μ). $\kappa_{0,ij}$ controls the thermal properties (e.g., thermal conductivity k). The specific functional forms of the weighting functions affect the numerical values of the fluid and particle properties, e.g., the fluid viscosity can be tuned by varying the functional forms of the weighting function w_d .

While there is a considerable flexibility in choosing the above parameters, the choice is subject to constraints by the level of resolution (related to ρ), numerical stability (related to σ_{ij} and functional forms of the weighting functions), computational efficiency (related to ρ), and, most importantly, the physical system the DPD system is set up to model. The physical system modeled by DPD is determined by the mapping between DPD and physical scales. The physical length and time corresponding to the unit length and time in DPD are typically denoted as $[L]$ and $[t]$, respectively. We showed earlier that, for fluctuating hydrodynamics simulations, where thermal fluctuations and momentum/thermal transport are both important, the following constraints must be satisfied:³¹

$$[t] = [L]^2 \nu / \bar{\nu}, \quad (12)$$

$$[t] = [L]^2 a / \bar{a}, \quad (13)$$

$$k_B T = \frac{\rho \nu^2 \bar{k}_B T}{\rho \nu^2 [L]}, \quad (14)$$

where ρ , ν , a , and $k_B T$ denote the density, kinematic viscosity, thermal diffusivity, and thermal energy of DPD fluids, respectively. The corresponding properties of real fluids are denoted by the same symbol but with an overline, e.g., $\bar{\rho}$ denotes the density of real fluids. Equations (12)–(14) set additional constraints for choosing DPD parameters. Equations (12) and (13) indicate that the Prandtl number of DPD fluids must match that of the real fluids, i.e., $\text{Pr} = \nu / a = \bar{\text{Pr}} = \bar{\nu} / \bar{a}$. Equation (14) indicates that the temperature of DPD fluids cannot be arbitrarily chosen. Instead, it depends on the length scale mapping (signified by $[L]$) and thermophysical properties of fluids. Since the thermophysical properties of DPD fluids depend on their temperature, the fluid temperature must be iteratively determined. In practice, one first determines the functional dependence of ν of DPD fluids on temperature and then finds the temperature that satisfies Eq. (14).

CODE VALIDATION

A code based on the model given by Eqs. (1)–(9) has been developed. Extensive tests of the code indicate that it can accurately model many canonical problems such as pressure driven flow in slit channels, spontaneous temperature fluctuations at mesoscopic scales, temperature evolution inside homogeneous solid slabs, and screening of electroosmotic flow by porous media.^{27,28} To test the capability of DPD in modeling thermal transport in strongly heterogeneous media, we study the enhancement in thermal conductivity by uniformly dispersing highly conducting nanoparticles inside a solid matrix, and the results obtained from the

DPD simulation agree well with the MG theory prediction.²⁸ To test the capability of DPD in modeling fluctuating hydrodynamics, we studied the Brownian diffusion of a single nanoparticle suspended in a box of fluids, and comparison of the diffusion coefficient obtained from the mean square displacement measured in DPD simulation agrees well with that predicted by the Stokes–Einstein relation.³¹ Here, we further test the capability of DPD in modeling fluctuating hydrodynamics focusing on the velocity autocorrelation function of Brownian particles.

We study the Brownian motion of a single nanoparticle suspended in a bulk of isothermal fluids at 300 K. The fluids have thermophysical properties very similar to those of water: $\bar{\rho} = 10^3 \text{ kg/m}^3$ and $\bar{\nu} = 0.89 \times 10^{-6} \text{ m}^2/\text{s}$, specific heat $c_p = 4.18 \text{ kJ/kg K}$, thermal conductivity $\bar{k} = 0.431 \text{ W/m K}$, and $\bar{\text{Pr}} = 8.64$. The nanoparticle has the same density and specific heat as the base fluids and a thermal conductivity that is 20 times higher than that of the base fluids. To build the DPD system, we build the nanoparticle by bonding 117 DPD beads together and model it as a rigid body. The nanoparticle is placed in a periodic simulation box measuring $18 \times 18 \times 18$ (in DPD unit). The rest of the simulation box is filled with DPD beads to produce a fluid density of $\rho = 6.0$. The weighting functions are taken as $w(r_{ij}/r_c) = 1 - r_{ij}/r_c$ and $w_d(r_{ij}/r_c) = \sqrt{1 - r_{ij}/r_c}$, and both functions are cut off at $r_{ij} > r_c$. The conservative interaction parameters are set as $a_{ff} = 10.0$ and $a_{fp} = 17.0$, where ff and fp denote fluid–fluid and fluid–nanoparticle interactions, respectively. σ_{ij} is taken as 5.0 for all the particle pairs. In order to model an intermediate-sized nanoparticle, we choose $[L] = 10 \text{ nm}$. Based on the above choices of parameters, we computed the kinematic viscosity of the DPD fluids, ν_{fluids} , as a function of temperature and a DPD temperature of $T = 0.3875$ is then chosen based on Eq. (14). Since we consider an isothermal scenario here, DPD parameters related to thermal transport are not important in the present simulation.

Figure 1(a) shows the scaled velocity autocorrelation function $\text{VACF}(t) = \langle \mathbf{v}(0) \cdot \mathbf{v}(t) \rangle / (3k_B T / m_{\text{np}})$ of the nanoparticle (m_{np} is the particle mass). We observe that $\text{VACF}(0)$ agrees with the theoretical prediction of 1.0 within statistical error. The VACF quickly decays as time increases. Importantly, at a later time, the VACF has a long-term tail that decays as $t^{-1.5}$, which agrees with prior theoretical and experimental studies of the dynamics of Brownian particles.³² Figure 1(b) shows the cross pair correlation between a nanoparticle and fluids. A slight density oscillation (typically called “layering”) of the fluids near the nanoparticle is observed. As in most particle simulations, the nonjump change in the pair correlation function makes the precise definition of the nanoparticle size difficult but we can still identify an effective particle radius of 1.65. Based on the mapping established earlier, i.e., $[L] = 10 \text{ nm}$, the nanoparticle studied here corresponds to a real nanoparticle with a diameter of 33 nm. We conclude that while a MD study of such nanoparticle is currently unrealistic, the fluctuating hydrodynamics behavior of such particle can be reasonably reproduced by DPD models. These simulation results, along with those dis-

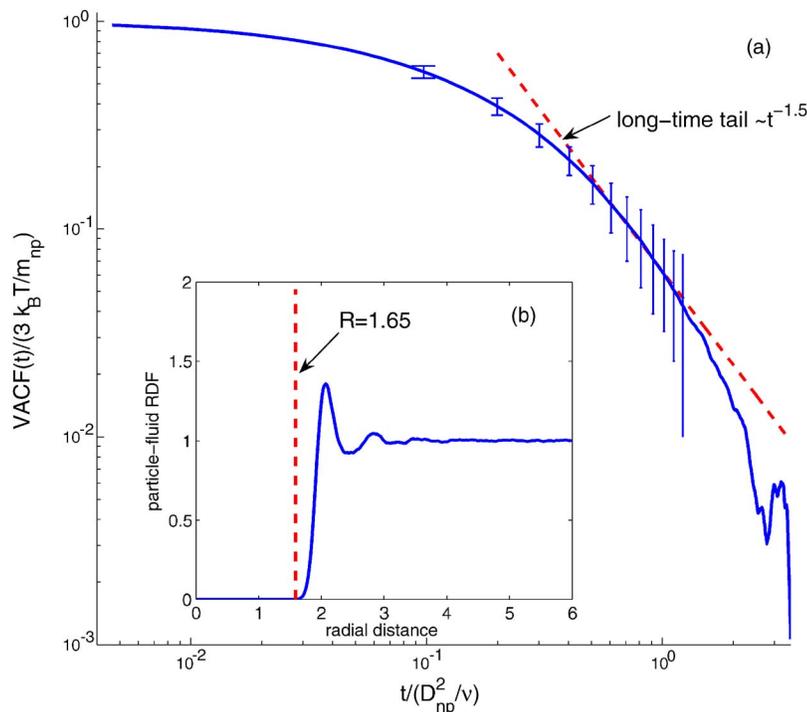


FIG. 1. (Color online) (a) Velocity autocorrelation function of a single nanoparticle suspended in a box of fluids. The error bars were computed from five independent runs. The time axis is scaled by D_{np}^2/ν , where D_{np} is the nanoparticle diameter. At a large time, the statistical error becomes comparable to the value of VACF and no reliable scaling behavior can be extracted from the data. (b) The nanoparticle-fluid pair correlation function.

cussed earlier, indicate that our DPD-based code can be reliably used to study fluctuating hydrodynamics problems in particulate suspensions.

HEAT CONDUCTION IN NANOFLUIDS

Simulation system and DPD parameters. The simulation systems consist of 5–20 nanoparticles dispersed in base fluids. The same nanoparticle and base fluids used in the above simulation are adopted here. As such, all the simulation parameters, e.g., a_{ij} and T , are the same as in the above simulation. In all the simulations, both the simulation box size ($24 \times 24 \times 24$) and the total number of beads (82 944, including the beads of nanoparticles and base fluids) are the same. Since the present simulations involve thermal transport, it is necessary to choose the relevant DPD parameters, i.e., c_v , $\kappa_{0,ij}$, and w_e . c_v can be determined by matching the total heat capacity of DPD fluids in a unit volume to that of the physical systems. Specifically, $\rho c_v = [L]^3 \overline{\rho c_p}$, or $c_v/k_B = c_p \rho [L]^3 / \rho k_B = 5.0313 \times 10^4$. Following convention, we set $w_e(r_{ij}/r_c) = 1 - r_{ij}/r_c$ for $r_{ij} < r_c$ and $w_e(r_{ij}/r_c) = 0$ otherwise. $\kappa_{0,ij}$ for the fluid-particle pairs, or $\kappa_{0,ff}$, is determined by requiring that $a_{\text{fluids}} = \nu_{\text{fluids}} / \text{Pr}_{\text{fluids}} = \nu_{\text{fluids}} / \overline{\text{Pr}_{\text{fluids}}}$. We tried a number of different $\kappa_{0,ff}$ and computed the corresponding thermal diffusivity of the DPD fluids, and $\kappa_{0,ff} = 6 \times 10^5$ was found to satisfy this requirement. $\kappa_{0,ij}$ between beads inside the nanoparticle, $\kappa_{0,pp}$, is chosen to be $20.0 \kappa_{0,ff}$, which ensures that the thermal conductivity of the nanoparticle is 20 times higher than that of the base fluids. $\kappa_{0,ij}$ between the base fluid beads and the beads of the nanoparticle, $\kappa_{0,pf}$, depends on the interfacial thermal resistance across the fluid-particle interface. Here, we assume that the interfacial thermal resistance is zero. To find the appropriate $\kappa_{0,pf}$ meeting this condition, we suspended one nanoparticle inside a box of fluids and impose a heat source inside the nanoparticle

and heat sink in a shell of fluids away from the nanoparticle. $\kappa_{0,pf}$ is tuned until the resultant temperature profile smoothly changes across the fluid-particle interface. Finally, $\kappa_{0,pf} = 5 \kappa_{0,ff}$ was obtained.

Calculation of thermal conductivity. To compute the thermal conductivity of the nanofluids (or base fluids), a non-equilibrium method widely used in MD simulations of thermal transport is used.¹¹ Figure 2 shows the schematic of the setup. Specifically, we extract a constant heat flux from a heat sink (width: 0.3) in the center of the simulation box and impose exactly the same amount of heat flux to a heat source (width: 0.3) at the edge of the simulation box. The resultant temperature profile is measured and fitted to a linear function. The thermal conductivity of the nanofluids is then computed by dividing the imposed heat flux by the calculated

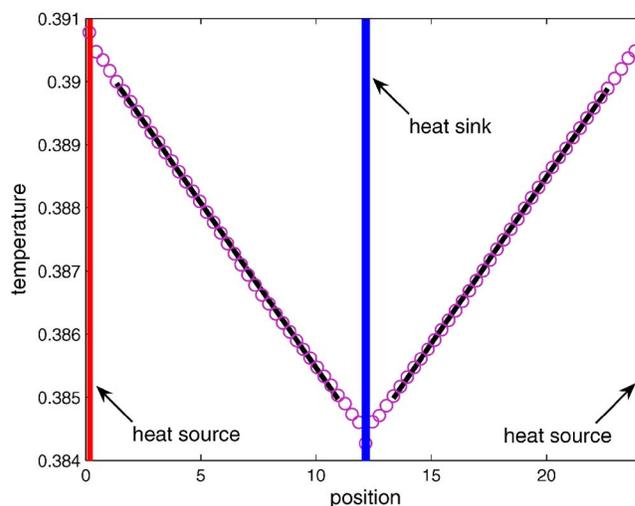


FIG. 2. (Color online) Schematic for calculating the thermal conductivity of nanofluids by using a nonequilibrium method. Note that the periodic boundary condition is used in all the three directions.

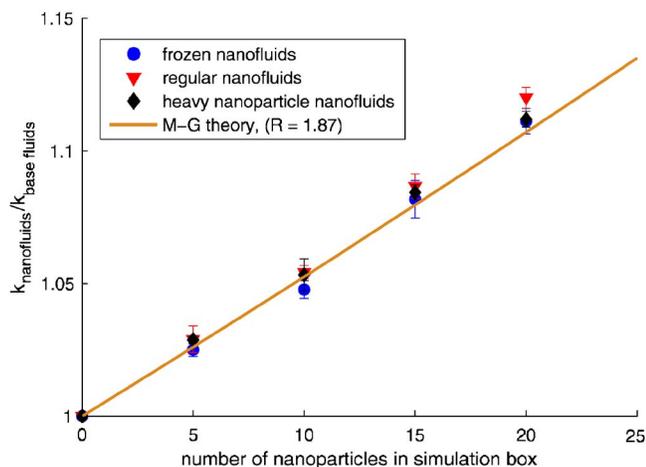


FIG. 3. (Color online) The enhancement of thermal conductivity of nanofluids compared to that of the base fluids obtained from DPD simulations. The size of the simulation box ($24 \times 24 \times 24$) is the same in all the simulations. If the radius of the nanoparticle is taken as 1.87, the volume fraction of the nanoparticle is 3.96% when there are 20 nanoparticles inside the simulation box.

temperature gradient. In all the simulations, the heat flux imposed is chosen to be small enough so that the maximum temperature difference inside the simulation box is less than 2% of the average temperature in the simulation system. Such a temperature difference is much smaller than typical nonequilibrium MD simulations employing the same method where a 20% difference in temperature is often reported.¹¹ The small temperature difference helps avoid the possible artifacts in simulation results due to the heterogeneity in fluid density induced by the temperature difference.

Results and discussion. For each nanoparticle loading, we consider three different scenarios: (1) both the nanoparticle and the base fluids are frozen to their original positions, (2) the base fluids freely move but the mass of the nanoparticles is increased to a hypothetical value of 10^4 to quench their Brownian motion, and (3) both the nanoparticle and the base fluids are allowed to freely move. We compute the thermal conductivity of the nanofluids for each scenario. To improve statistical accuracy, each simulation is repeated five times with different initial configurations and clustering of nanoparticles is not observed in any of the simulations. Figure 3 compares the enhancement of thermal conductivity as a function of the number of nanoparticles inside the simulation box. We observe that within statistical error, the enhancement of thermal conductivity in the three different scenarios is the same. We also find that the calculated enhancement can be fitted to the MG theory by assuming a particle radius of $R_p=1.87$. This radius is about 10% larger than the effective particle radius obtained earlier from the particle-fluid pair correlation function. However, given the ambiguity in determining the size of the nanoparticle, we conclude that the agreement with the MG theory is good. Note that the ambiguity in particle size, although rarely mentioned, exists in all particle simulations (e.g., MD simulation) and can be best minimized by increasing the size of the nanoparticle, which increases the computational cost. The above results indicate that the Brownian motion of nanoparticles does not play an

important role in enhancing the thermal conductivity of nanofluids, and for the well-dispersed nanofluids with spherical particles studied here, the classical effective medium theory can predict the thermal transport well. The absence of detectable effects of nanoparticle Brownian motion on the thermal transport in nanofluids has been theoretically explained. One of the important reasons, which has been neglected by many researchers, is that although nanoparticles induce microflows, the net flow generated by a *randomly* moving nanoparticle cannot propagate to a position far from the particle to enhance the thermal transport. A more comprehensive discussion on this topic can be found in Refs. 14 and 33, and the above simulations provide new evidence supporting the conclusion of these theories.

We note that our simulation results are not necessarily in contradiction with some of the recent MD simulations showing that the enhancement of the thermal conductivity in nanofluids can exceed that predicted by the MG theory.^{15,34} In these simulations, the transport processes are probed at atomistic scales and the observed anomalous enhancements in the thermal conductivity are typically attributed to atomistic effects (e.g., significant potential energy fluctuations near nanoparticles) confined within a few angstroms near nanoparticles, which are not modeled in our mesoscopic simulations. As the nanoparticle diameter increases to tens of nanometers, the contribution of such atomistic effects will become less dominant. It is thus desirable to know whether an anomalous thermal conductivity enhancement can be observed in the absence of such atomistic effects. While this is beyond the capability of current MD simulations, our simulations provide direct evidence that the anomalous thermal conductivity enhancement is unlikely.

CONCLUSIONS

In summary, we investigate the heat conduction in nanofluids by using the energy-conserving DPD method. By using such a method, we effectively solve the fluctuating hydrodynamics equations governing the motion of base fluids/nanoparticle and the thermal transport in particulate suspensions, and the fluctuation-dissipation theorem is rigorously satisfied. Compared to previously reported atomistic simulations of nanofluids, the atomistic details of thermal/momentum transport are not explicitly resolved in our simulation due to coarse graining. However, the coarse graining enables us to probe the transport processes at much larger length scales that more closely resemble that in engineering practice. Our simulations indicate that the Brownian motion of nanoparticles play a negligible role in determining the thermal conductivity of nanofluids at least within the framework of fluctuating hydrodynamics at mesoscales.

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