Given the matrix \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \):

1. (25 Pts.) Calculate the inverse of \( A \).
2. (20 Pts.) Form the eigenvalue problem associated with \( A \).
3. (20 Pts.) Solve for the eigenvalues.
4. (20 Pts.) Form the matrix \( A^2 \) and calculate its eigenvalues.
5. (15 Pts.) Demonstrate that
   a. The sum of the eigenvalues of \( A \) equals the sum of the diagonal entries of \( A \).
   b. The product of the eigenvalues of \( A \) equals the determinant of \( A \).
   c. The eigenvalues of \( A^2 \) equal the squares of the eigenvalues of \( A \).