Figure 1 shows a simplified model of one of the methods used for silicon crystal growth. For $x < -L$, a long crucible (heat pipe) is maintaining silicon in the molten state at $T_m$ (silicon melting temperature). For $x > 0$ the silicon is cooled. The region $-L < x < 0$ is kept adiabatic. The objective of the process is to adjust the velocity by which the melt is fed into the system such that the solid-liquid interface is located within the adiabatic zone. In order to estimate the value for the maximum allowable velocity for these conditions (interface located at $x = 0$), model the above process by neglecting radial temperature gradients in the silicon (i.e. use the fin approximation).

(i) Draw a control volume and determine the governing equations for both the adiabatic zone ($-L < x < 0$), and for $x > 0$, assuming that the solid-liquid interface is located at $x = 0$. (50%)

Consider steady-state. Assume that the thermophysical properties (including the density $\rho$) of the solid and the liquid are identical. Beside the symbols given in Fig. 1, the cross sectional area is $A$, perimeter of the cross section is $P$, heat capacity per unit mass is $c_p$, the thermal conductivity is $k$.

For $-L < x < 0$, consider heat conduction and advection.

For $x > 0$, consider heat conduction, advection and convection.

The boundary conditions are:

$x = -L$ \hspace{1cm} $T = T_m$

$x = 0$ \hspace{1cm} $T = T_m$

$x = 0$ \hspace{1cm} $-kA \frac{\partial T}{\partial x} \bigg|_{x=0}^- + kA \frac{\partial T}{\partial x} \bigg|_{x=0}^+ + \rho V A h_{fs} = 0$

$x \to \infty$ \hspace{1cm} $T$ is finite

where $h_{fs}$ is the enthalpy of solidification.

(i) Solve the governing equations for the temperature distribution in each zone. (25%)

(ii) Determine the velocity required for the interface to be located at $x = 0$. (15%)

(iii) Describe qualitatively how the solid-liquid interface would move if the velocity is lower than this critical value, and what would happen if it is higher. (10%)